

NAME :

DATE :

MARKS :

# Quadratic Equations by completing square

## Learning Objectives

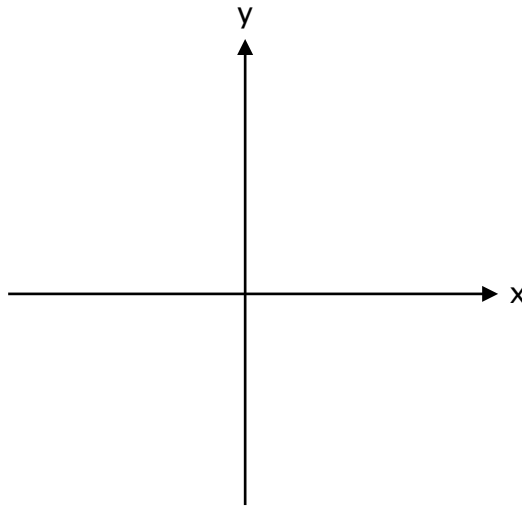
Students should be able to :

- (a) Derive and solve quadratic equations by completing the square
- (b) Find turning points of quadratics by completing the square.

Nos	Questions	Reference
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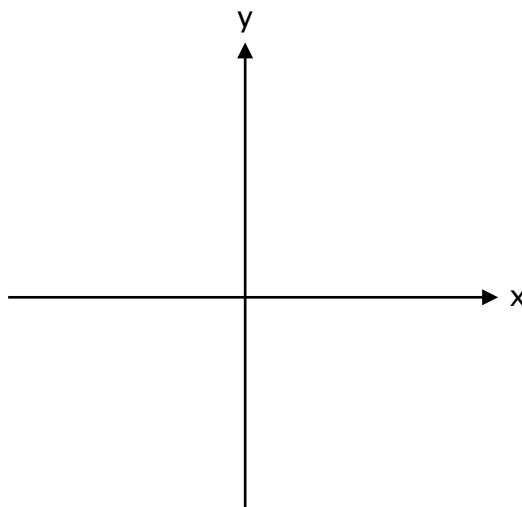
1 (a) Express  $x^2 - 6x + 2$  in the form  $(x - a)^2 + b$ .

(b) Sketch the graph of  $y = x^2 - 6x + 2$ , showing clearly the coordinates of the turning point.



2 (a) Express  $y = x^2 + 3x + 2$  in the form of  $y = (x + p)^2 - q$ .

(b) Sketch the graph of  $y = x^2 + 3x + 2$ , indicating clearly its intercepts with the axes and the turning point.

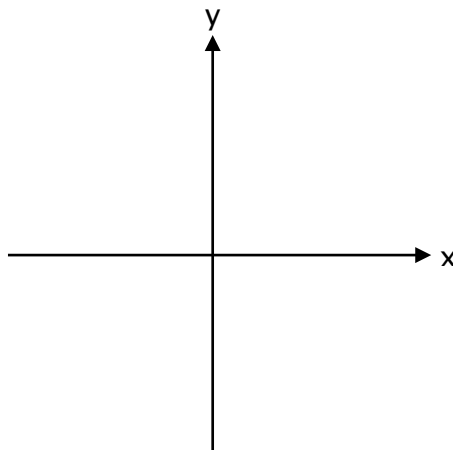


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- 3 (a) Express  $x^2 - 8x - 29$  in the form  $(x - a)^2 - b$ .
- (b) Hence, solve the equation  $x^2 - 8x - 29 = 0$ , giving your answers correct to 2 decimal places.

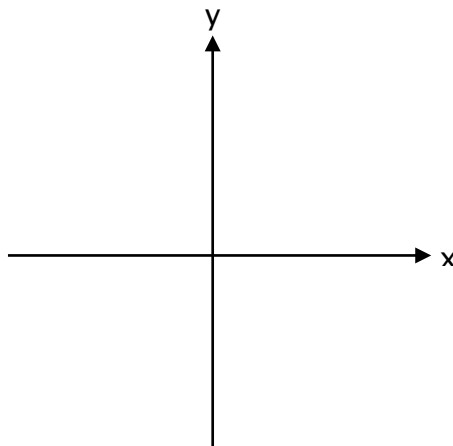
- 4 (a) Express  $x^2 - 3x - 5$  in the form  $(x - a)^2 - b$ .
- (b) Hence, solve the equation  $x^2 - 3x - 5 = 0$ , giving your answers correct to 2 decimal places.

- 5 (a) Sketch the graph of  $y = 2 - (x + 3)^2$ .



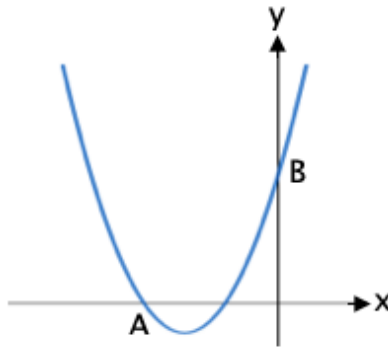
- (b) Write down the equation of the line of symmetry of  $y = 2 - (x + 3)^2$ .

- (c) Sketch the graph of  $y = (x + 3)(x - 1)$ .

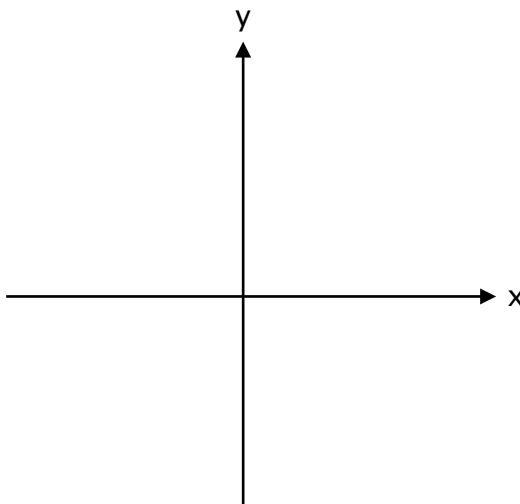


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- 6 The figure below shows a quadratic graph  $y = x^2 + 7x + 10$ . The graph cuts the  $x$  – axis at point A and the  $y$  – axis at point B.



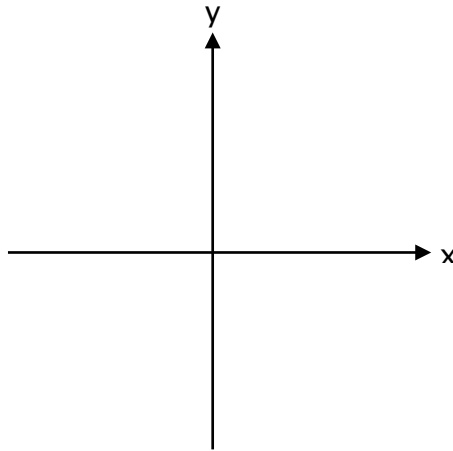
- (a) Find the coordinates of A and B.
- (b) State the minimum point of the graph.
- 7 (a) Express  $y = -x^2 + 4x + 5$  in the form  $(x - h)^2 + k$ .
- (b) Sketch the graph of  $y = -x^2 + 4x + 5$ , showing the intercepts with the axes clearly.



- (c) Write down the equation of the line of symmetry of  $y = -x^2 + 4x + 5$ .

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- 8 (a) Sketch the graph of  $y = 5 - (x + 2)^2$ . Label the  $y$  – intercept clearly.



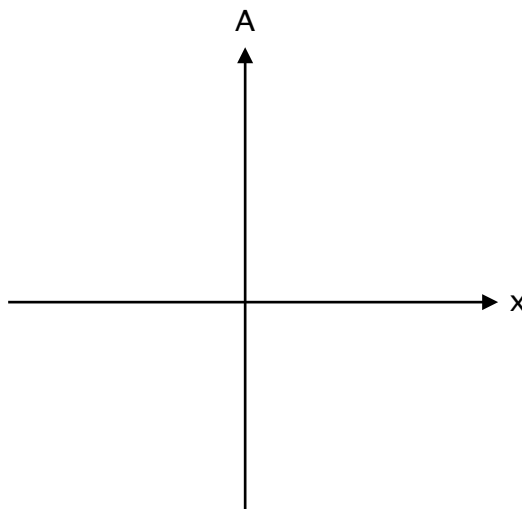
- (b) Write down the coordinates of the maximum point of the curve of  $y = 5 - (x + 2)^2$ .

- 9 The area of a rectangle,  $A \text{ cm}^2$ , is given by  $A = x^2 - 4x + 7$ .

- (a) Express  $y = x^2 - 4x + 7$  in the form  $(x - a)^2 + b$ .

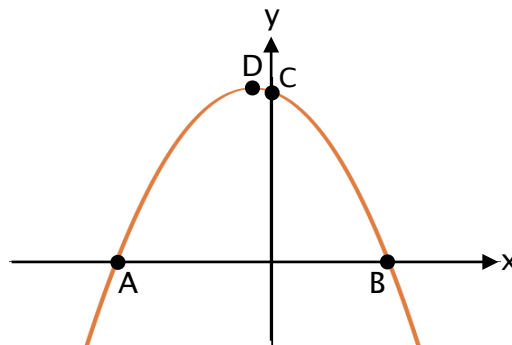
- (b) State the minimum area of the rectangle.

- (c) Sketch the graph of  $A = x^2 - 4x + 7$  on the axes below, clearly stating the turning point and the vertical intercept.



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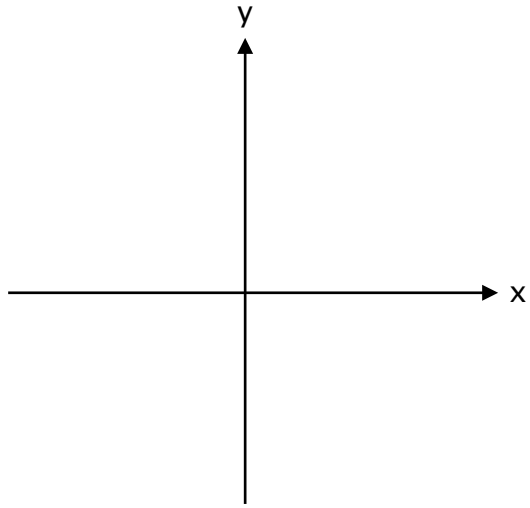
- 10 (a) Express  $x^2 - 4x - 1$  in the form  $(x - a)^2 - b$ .
- (b) Hence, solve the equation  $x^2 - 4x - 1 = 0$ , giving your answers correct to 2 decimal places.
- 11 (a) Express  $x^2 - 6x - 1$  in the form  $(x - p)^2 + q$ .
- (b) Hence, solve the equation  $x^2 - 6x - 1 = 0$ , giving your answers correct to 2 decimal places.
- 12 (a) Express  $x^2 - 5x - 8$  in the form  $(x + a)^2 + b$ .
- (b) Hence, solve the equation  $x^2 - 5x - 8 = 0$ , giving your answers correct to 2 decimal places.
- 13 The curve  $y = 6 - x - 2x^2$  cuts the  $x$  - axis at points A and B and the  $y$  - axis at C. It has a maximum turning point at D.



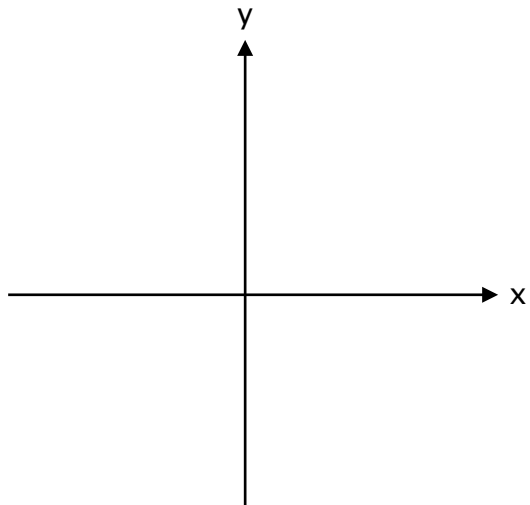
- (a) Calculate the coordinates of A, B and C.
- (b) Hence, or otherwise,
- write down the equation of the line of symmetry,
  - find the maximum value of  $y$ .

Nos	Questions	Reference
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- 14 Sketch the graph of  $y = -(2 - x)^2 + 1$  on the axes below.  
Indicate clearly the values where the graph crosses the x – and y – axes and the coordinates of any turning points.



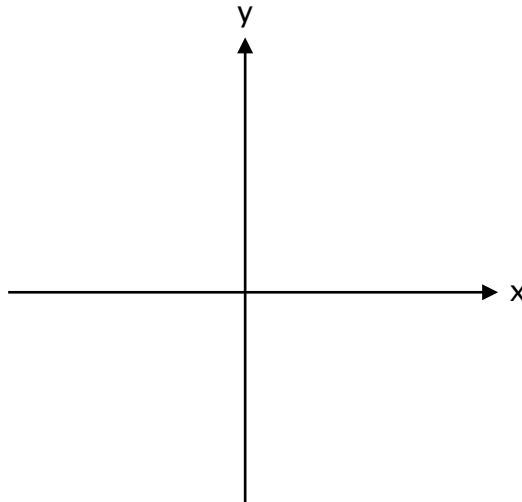
- 15 Sketch the graph of  $y = (x + 2)(4 - x)$  on the axes below.  
Indicate clearly the x – intercepts, y – intercept and the coordinates of the turning points.



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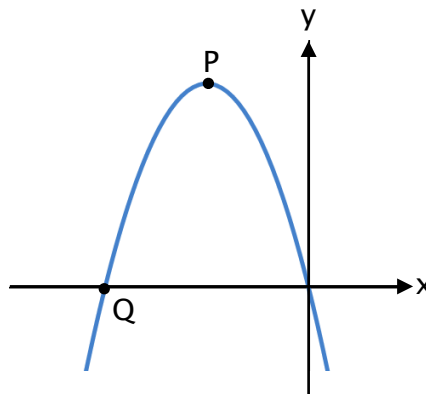
16 (a) Express  $x^2 - \frac{1}{4}x$  in the form  $(x - b)^2 + c$ .

(b) Sketch the graph of  $y = \frac{1}{4}x - x^2$ .



(c) Find the coordinates of the maximum point of  $y = \frac{1}{4}x - x^2$ .

17 The graph of  $y = 4 - (x + 2)^2$  is shown below.



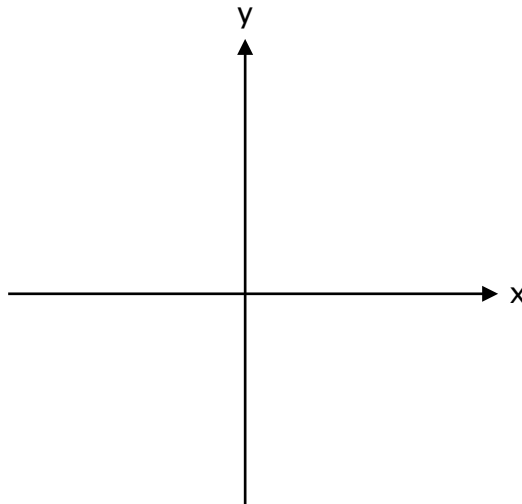
- State the coordinates of the maximum point P.
- State the coordinates of the point Q where the graph cuts the x – axis.
- Write down the equation of the line of symmetry of the graph.
- State the largest value of a such that  $4 - (x + 2)^2 = a$  has real solutions.



Nos	Questions	Reference
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18 (a) Express the function  $y = -x^2 + 8x - 5$  in the form  $y = -(x - h)^2 + k$ .

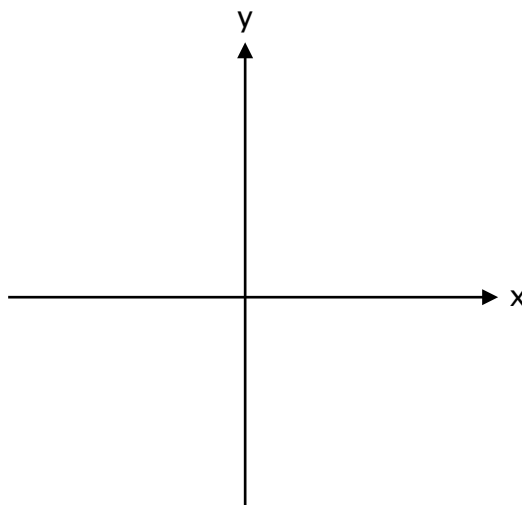
(b) Sketch the graph of  $y = -x^2 + 8x - 5$ . Label the y – intercept and turning point.



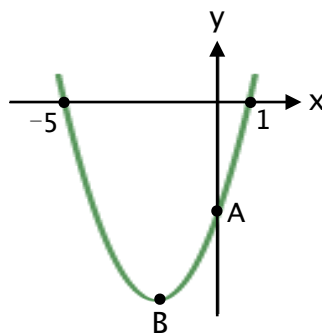
(c) Hence, or otherwise, solve the equation  $-x^2 + 8x - 5 = -10$ .

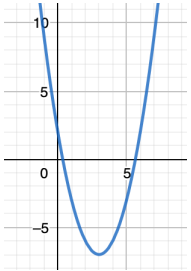
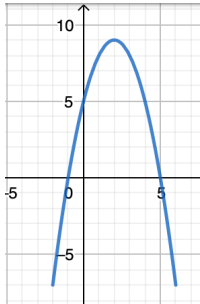
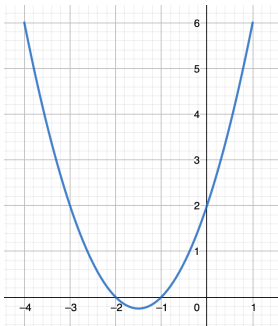
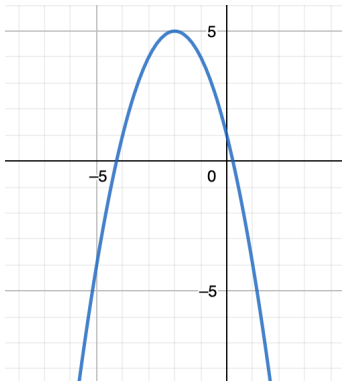
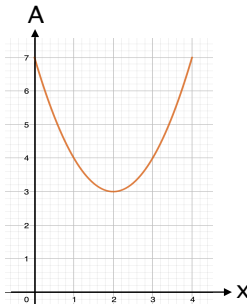
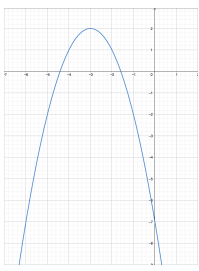
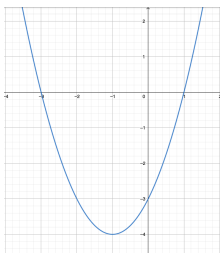
19 (a) Express  $-x^2 - 5x - 6$  in the form  $-(x + a)(x + b)$ , where a and b are constants.

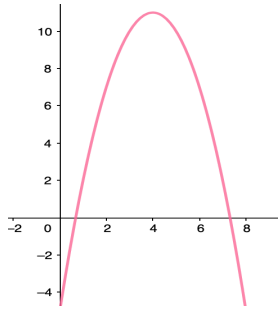
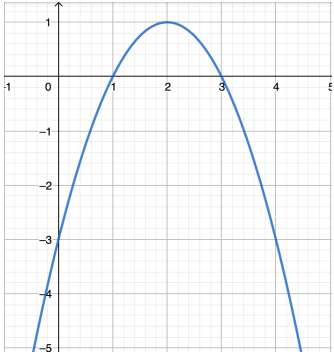
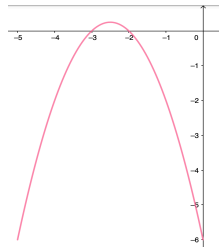
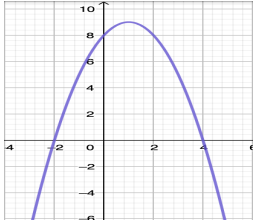
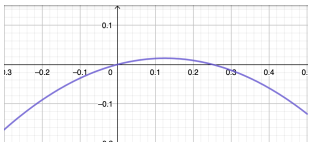
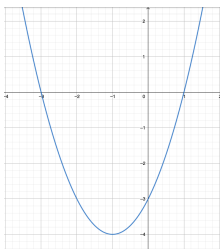
(b) Hence, Sketch the curve of  $y = -x^2 - 5x - 6$ , indicating clearly the intercepts and turning point.



Nos	Questions	Reference
20	<p>The equation of a curve is <math>y = x^2 + bx + c</math> where <math>b</math> and <math>c</math> are constants.</p> <p>(a) Given that <math>(2, 0)</math> is a point on the curve, show that <math>b = -\frac{4+c}{2}</math>.</p> <p>(b) If the <math>y</math> – intercept of the curve is 14, find the values of <math>b</math> and <math>c</math>.</p>	
21	Express $-8x - 11 + x^2$ in the form $(x + p)^2 + q$ .	
22	<p>(a) <math>(-5, 2)</math> the maximum point of a quadratic curve.          Write the equation of the graph in the form <math>y = p - (x + p)^2</math>.</p> <p>(b) A straight line on the <math>xy</math> – axes passes through <math>(-5, 2)</math> and cuts the <math>x</math> – axis at <math>x = 1</math>.          Find the equation of the straight line.</p>	
23	<p>The curve cuts the <math>x</math> – axis at <math>-5</math> and <math>1</math> and the <math>y</math> – axis at <math>A</math>. <math>B</math> is the minimum point on the curve.          Express the equation of the curve in the form of <math>y = a(x + h)^2 - 18</math>, where <math>a</math> and <math>h</math> are constants.</p>	



Nos	Questions	Nos	Questions
1	(a) $(x - 3)^2 - 7$ (b) 	7	(a) $-(x - 2)^2 + 9$ (b) 
2	(a) $(x + \frac{3}{2})^2 - \frac{1}{4}$ (b) 	8	(a) 
3	(a) $(x - 4)^2 - 45$ (b) 10.708 or -2.708	9	(a) $(x - 2)^2 + 3$ (b) Minimum area = 3 cm <sup>2</sup>
4	(a) $(x - \frac{3}{2})^2 - \frac{29}{4}$ (b) 4.19 or -1.19		(c) 
5	(a)  (b) $x = -3$ (c) 	10	(a) $(x - 2)^2 - 5$ (b) 4.24 or -0.24
6	(a) A = (-5, 0) B = (0, 10) (b) Minimum point = $(-\frac{7}{2}, -\frac{9}{4})$	11	(a) $(x - 3)^2 - 10$ (b) 6.16 or -0.162
		12	(a) $(x - \frac{5}{2})^2 - \frac{57}{4}$ (b) 6.27 or -1.27

Nos	Questions	Nos	Questions
13	(a) $A = (-2, 0)$ $B = (1\frac{1}{2}, 0)$ $C = (0, 6)$ (b) $x = -\frac{1}{4}$ (ii) $\text{Max } y = 6\frac{1}{8}$	18	(a) $-(x - 4)^2 + 11$ (b)  (c) 8.58 or -0.583
14		19	(a) $-(x + 3)(x + 2)$ (b) 
15		20	(b) $b = -9; c = 14$
16	(a) $(x - \frac{1}{8})^2 - \frac{1}{64}$ (b)  (c) $(\frac{1}{8}, \frac{1}{64})$	21	$(x - 4)^2 - 27$
17	(a) $P = (-2, 4)$ (b) $Q = (-4, 0)$ (c) $x = -2$ (d) $a = 4$ (c) 	22	(a) $2 - (x + 5)^2$ (b) $y = \frac{1}{3}x - \frac{1}{3}$
		23	$2(x + 2)^2 - 18$