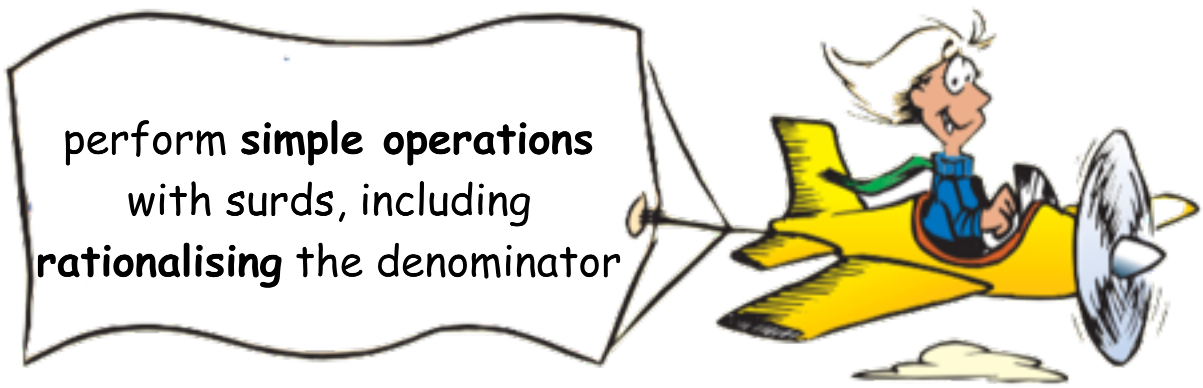
Surds

Learning Objectives

Students should be able to



Nos	Questions	Reference
1	Express $\dfrac{4m\sqrt{m} - \dfrac{9}{\sqrt{m}}}{2\sqrt{m} + \dfrac{3}{\sqrt{m}}}$ in the form Am + B, where A and B are integers to be found. [3]	Q2/0606/13/O/N/16

$$\frac{4m\sqrt{m} - \frac{9}{\sqrt{m}}}{2\sqrt{m} + \frac{3}{\sqrt{m}}} = \frac{4m\sqrt{m} - \frac{9}{\sqrt{m}}}{2\sqrt{m} + \frac{3}{\sqrt{m}}} \times \frac{2\sqrt{m} - \frac{3}{\sqrt{m}}}{2\sqrt{m} - \frac{3}{\sqrt{m}}} = \frac{8m^2 - 12m - 18 + \frac{27}{m}}{4m - \frac{9}{m}} = \frac{(4m - \frac{9}{m})(2m - 3)}{4m - \frac{9}{m}} = 2m - 3$$

OR

$$(2\sqrt{m} + \frac{3}{\sqrt{m}})(Am + B) = 2Am\sqrt{m} + 2B\sqrt{m} + \frac{3Am}{\sqrt{m}} + \frac{3B}{\sqrt{m}} = 4m\sqrt{m} - \frac{9}{\sqrt{m}}$$

2Am = 4m => 2A = 4 => A = 2 and 3B = -9 => 3B = -9 => B = -3

Nos	Questions	Reference
2	**Do not use a calculator in this question.** Find the positive value of x for which $(4 + \sqrt{5})x^2 + (2 - \sqrt{5})x - 1 = 0$, giving your answer in the form $\dfrac{a + \sqrt{5}}{b}$, where a and b are integers. [6]	Q4/0606/12/M/J/16

$$(4 + \sqrt{5})x^2 + (2 - \sqrt{5})x - 1 = 0$$

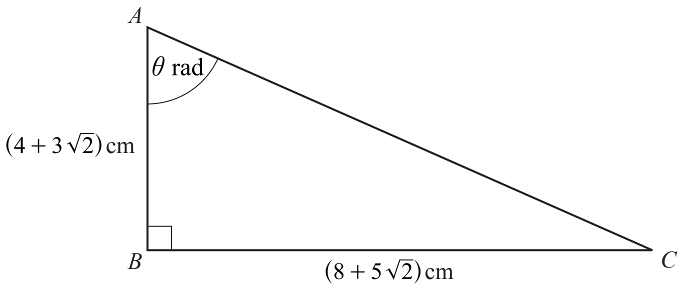
$$x = \frac{-(2 - \sqrt{5}) \pm \sqrt{(2 - \sqrt{5})^2 - 4(4 + \sqrt{5})(-1)}}{2(4 + \sqrt{5})} = \frac{-(2 - \sqrt{5}) \pm \sqrt{4 - 4\sqrt{5} + 5 + 16 + 4\sqrt{5}}}{2(4 + \sqrt{5})}$$

$$x = \frac{3 + \sqrt{5}}{8 + 2\sqrt{5}} \text{ or } \frac{-7 + \sqrt{5}}{8 + 2\sqrt{5}} \text{ (rejected as -ve value)}$$

$$x = \frac{3 + \sqrt{5}}{2(4 + \sqrt{5})} \cdot \frac{4 - \sqrt{5}}{4 - \sqrt{5}} = \frac{12 + \sqrt{5} - 5}{2(16 - 5)} = \frac{7 + \sqrt{5}}{22}$$

Nos	Questions	Reference

4 **Do not use a calculator in this question.**



The diagram shows the triangle ABC where angle B is a right angle, AB = $(4 + 3\sqrt{2})$ cm, BC = $(8 + 5\sqrt{2})$ cm and angle BAC = θ radians. Showing all your working, find

(i) tan θ in the form $a + b\sqrt{2}$, where a and b are integers, [2]

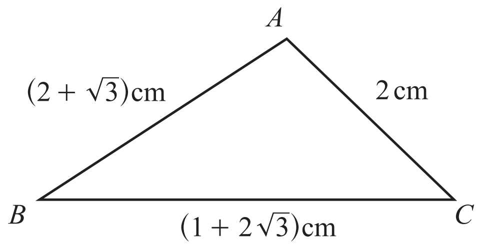
(ii) $\sec^2\theta$ in the form $c + d\sqrt{2}$, where c and d are integer. [3]

Q2/0606/11/M/J/15
Q2/0606/13/M/J/15

(i) $\tan\theta = \dfrac{8 + 5\sqrt{2}}{4 + 3\sqrt{2}} \cdot \dfrac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}} = \dfrac{2 - 4\sqrt{2}}{16 - 18} = 2\sqrt{2} - 1$

(ii) $\sec^2\theta = 1 + \tan^2\theta = 1 + (2\sqrt{2} - 1)^2 = 10 - 4\sqrt{2}$

5 The diagram shows a triangle ABC such that AB = $(2 + \sqrt{3})$ cm, BC = $(1 + 2\sqrt{3})$ cm and AC = 2 cm.



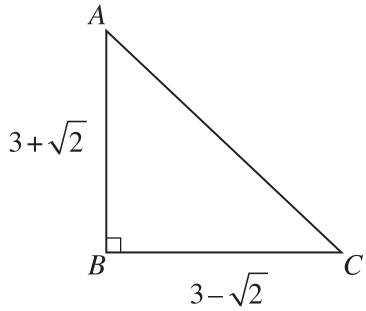
Without using a calculator, find the value of cosA in the form $a + b\sqrt{3}$, where a and b are constants to be found. [4]

Q10/0606/13/O/N/14

$$\cos A = \frac{(2 + \sqrt{3})^2 + 2^2 - (1 + 2\sqrt{3})^2}{2(2 + \sqrt{3})(2)} = \frac{-1}{2(2 + \sqrt{3})} \cdot \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} = \frac{-2 + \sqrt{3}}{2} = -1 + \frac{\sqrt{3}}{2}$$

Nos	Questions	Reference

13



The diagram shows a triangle ABC, where angle B is a right angle, the length of AB = $3 + \sqrt{2}$ and the length of BC = $3 - \sqrt{2}$.

(i) Find the length of AC in the form $\sqrt{k}$, where k is an integer. [2]

(ii) Find tan A in the form $\frac{a + b\sqrt{2}}{c}$, where a, b and c are integers. [3]

Q5/0606/01/O/N/09

(i) $AC = \sqrt{(3 + \sqrt{2})^2 + (3 - \sqrt{2})^2} = \sqrt{22}$

(ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}} = \frac{11 - 6\sqrt{2}}{7}$

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Express $\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [3]

Q1/0606/01/M/J/08

$$\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = \frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} \cdot \frac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}} = \frac{50 - 36\sqrt{2}}{-2} = -25 + 18\sqrt{2}$$

15

Given that $p = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, express in its simplest surd form,

(i) p, [3]

(ii) $p - \frac{1}{p}$. [2]

Q3/0606/01/O/N/07

(i) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

(ii) $p - \frac{1}{p} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} - \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{4 + 2\sqrt{3} - 4 + 2\sqrt{3}}{2} = 2\sqrt{3}$